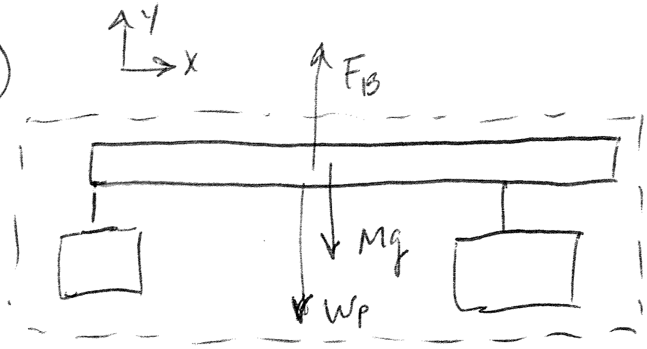
1



$$\Sigma F_y: \quad F_B - Mg - W_p = 0$$

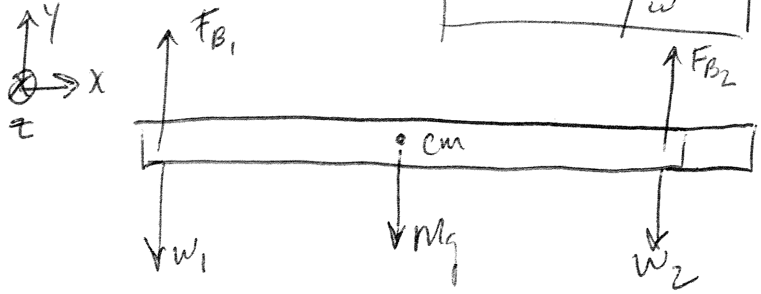
$$F_B = \frac{1}{2}\rho_w V g$$

$$W_p = \frac{1}{4}\rho_w V g$$

$$\Sigma F_y: \quad \frac{1}{2}\rho_w V g - \frac{1}{4}\rho_w V g = Mg$$

$$\frac{1}{4}\rho_w V \cancel{g} = M\cancel{g}$$

$$\boxed{V = \frac{4M}{\rho_w}}$$



$$\Sigma \tau_z: \quad \left(\rho_w \frac{1}{2} V_1 g\right)\left(\frac{L}{2}\right) - \left(\frac{1}{4}\rho_w V_1 g\right)\left(\frac{L}{2}\right) - \left(\rho_w \frac{1}{2} V_2 g\right)\left(\frac{L}{4}\right)$$

$$+ \left(\frac{1}{4}\rho_w V_2 g\right)\left(\frac{L}{4}\right) = 0$$

$$\frac{1}{4}\rho_w V_1 g - \frac{1}{8}\rho_w V_1 g - \frac{1}{8}\rho_w V_2 g + \frac{1}{16}\rho_w V_2 g = 0$$

$$\frac{1}{8}\rho_w V_1 g = \frac{1}{16}\rho_w V_2 g$$

$$V_1 = \frac{1}{2}V_2$$

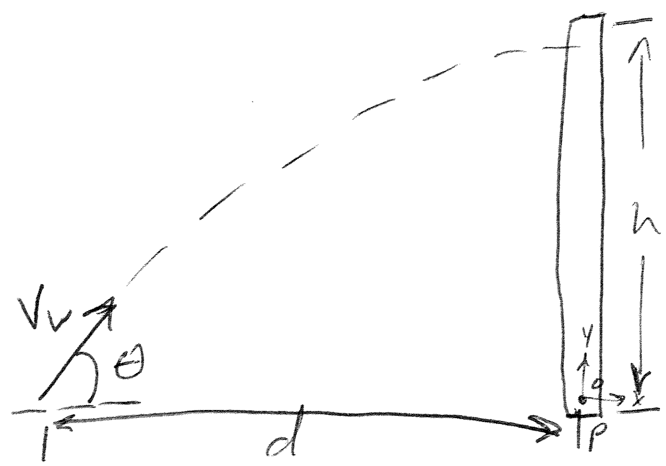
$$V_1 + V_2 = V = \frac{4M}{\rho_w}$$

$$V_1 + 2V_1 = \frac{4M}{\rho_w}$$

$$3V_1 = \frac{4M}{\rho_w}$$

$$\boxed{\begin{aligned} V_1 &= \frac{4}{3}\frac{M}{\rho_w} \\ V_2 &= \frac{8}{3}\frac{M}{\rho_w} \end{aligned}}$$

(2)



$$x(t) = -d + v_L \cos\theta\, t \qquad y(t) = v_L \sin\theta\, t - \frac{1}{2} g t^2$$

$$v_x(t) = v_L \cos\theta \qquad v_y(t) = v_L \sin\theta - g t$$

@ target $t = T$ $\quad v_y(T) = 0 \quad x(T) = 0 \quad y(T) = h$

$$v_y(T) = 0 = v_L \sin\theta - gT$$

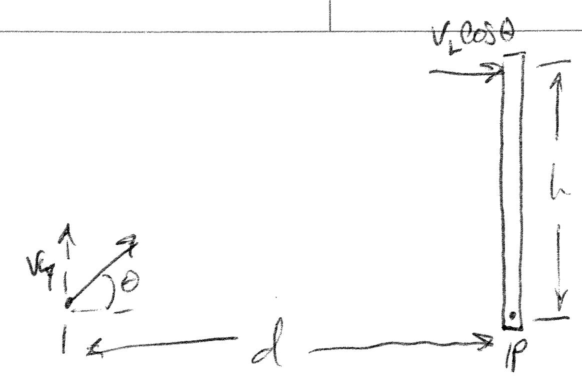
$$T = \frac{v_L \sin\theta}{g}$$

$$y(T) = h = v_L \sin\theta \left(\frac{v_L \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{v_L \sin\theta}{g}\right)^2$$

$$h = \frac{v_L^2}{g} \sin^2\theta - \frac{1}{2} \frac{v_L^2}{g} \sin^2\theta$$

$$h = \frac{1}{2} \frac{v_L^2}{g} \sin^2\theta$$

$$\sin^2\theta = \frac{2gh}{v_L^2} \qquad \boxed{\theta = \sin^{-1}\left[\sqrt{\frac{2gh}{v_L^2}}\right]}$$



$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = r_{\perp} m v$$

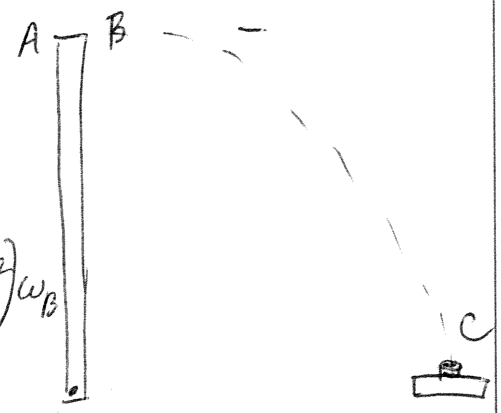
$$L_{throw} = d m v_L \sin\theta$$

$$L_{target} = h m v_L \cos\theta$$

if $d = h$

$$\boxed{\frac{L_{throw}}{L_{target}} = \frac{\cancel{dm}v_L \sin\theta}{\cancel{dm}v_L \cos\theta} = \tan\theta}$$

no external torques so



$$L_A = L_B$$

$$L_A = L_{target} = h m v_L \cos\theta$$

$$L_B = I\omega_B = \left(\tfrac{1}{12}Mh^2 + M\left(\tfrac{h}{2}\right)^2 + mh^2\right)\omega_B$$

$$h m v_L \cos\theta = \left(\tfrac{1}{3}M + m\right)h^2 \omega_B$$

$$\omega_B = \frac{m v_L \cos\theta}{\left(\frac{1}{3}M + m\right)h}$$

$$E_B = E_C$$

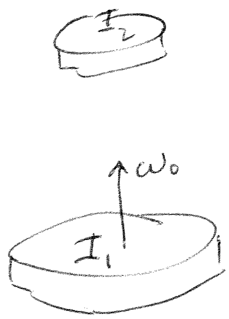
$$E_B = \frac{1}{2}\left(\frac{1}{3}Mh^2 + mh^2\right)\omega_B^2 + Mg\frac{h}{2} + mgh$$

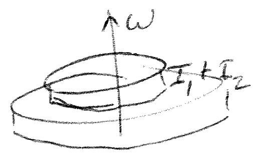
$$E_C = \frac{1}{2}\left(\frac{1}{3}Mh^2 + mh^2\right)\omega_C^2$$

$$(Mh^2 + 3mh^2)\,\omega_C^2 = (Mh^2 + 3mh^2)\,\omega_B^2 + 3Mgh + 6mgh$$

$$\boxed{\omega_C^2 = \omega_B^2 + \frac{3(Mgh + 2mgh)}{(Mh^2 + 3mh^2)}}$$

(4)





$L_i = I_1 \omega_0$ $\qquad$ $L_f = (I_1+I_2)\omega$

$E_i = \frac{1}{2} I_1 \omega_0^2$ $\qquad$ $E_f = \frac{1}{2}(I_1+I_2)\omega^2$

$L_i = L_f$

$I_1\omega_0 = (I_1+I_2)\omega \longrightarrow \omega = \dfrac{I_1}{I_1+I_2}\omega_0$

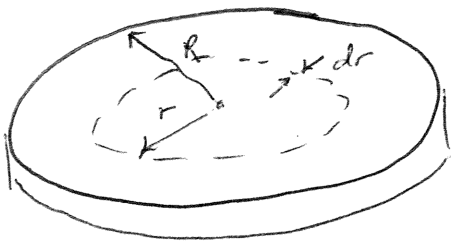
$E_i \stackrel{?}{=} E_f$

$\frac{1}{2} I_1 \omega_0^2 \stackrel{?}{=} \frac{1}{2}(I_1+I_2)\left[\dfrac{I_1}{I_1+I_2}\omega_0\right]^2$

$\frac{1}{2} I_1 \omega_0^2 \neq \frac{1}{2}\dfrac{I_1^2}{I_1+I_2}\omega_0^2$ Energy is lost to dissipative force of friction.

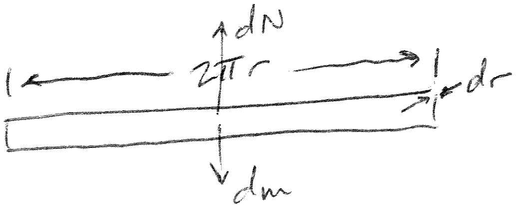
$\dfrac{E_i}{E_f} = \dfrac{\cancel{\frac{1}{2}} \cancel{I_1} \cancel{\omega_0^2}}{\cancel{\frac{1}{2}} \dfrac{I_1^{\cancel{2}}}{I_1+I_2}\cancel{\omega_0^2}} = \dfrac{I_1+I_2}{I_1}$ $\qquad$ $\boxed{\dfrac{E_f}{E_i} = \dfrac{I_1}{I_1+I_2}}$

Torque is due to friction between disks



Area density $= \frac{M}{\pi R^2} = \sigma$

$$dN = dm = \sigma\, 2\pi r\, dr$$



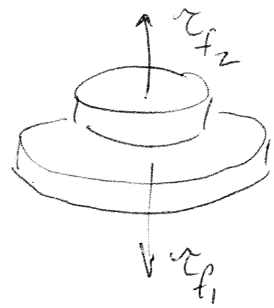
$$df = \mu_k\, dN = \mu_k \sigma g 2\pi r\, dr$$

$$d\tau = (\mu_k \sigma g 2\pi r\, dr)\, r$$

$$\tau = \int_0^R \mu_k \frac{M}{\cancel{\pi} R^2} g 2\cancel{\pi}\; r^2\, dr$$

$$\boxed{\tau_f = \frac{2\mu_k M g}{\cancel{R^2}} \frac{R^{\cancel{3}}}{3} = \frac{2}{3}\mu_k M R g}$$

$$\Sigma\tau_1 = \tau_{f_1} = -\frac{2}{3}\mu_k M R g = I_1(-\alpha_1)$$



$$\alpha_1 = \frac{2\mu_k M R g}{3 I_1}$$

$$\omega_1(t) = \omega_0 - \alpha_1 t$$

@ $t = T$ $\quad \omega_1(T) = \omega = \frac{I_1}{I_1 + I_2}\omega_0$

$$\frac{I_1}{I_1 + I_2}\omega_0 = \omega_0 - \frac{2\mu_k M R g}{3 I_1} T$$

$$-\frac{2\mu_k M R g}{3I_1}T = \omega_0\left(\frac{I_1}{I_1+I_2}-1\right) = \omega_0\left(\frac{\cancel{I_1}-\cancel{I_1}-I_2}{I_1+I_2}\right)$$

$$\boxed{T_1 = \frac{3I_1 I_2\omega_0}{2\mu_k M R g(I_1+I_2)}}$$

$$\Sigma\tau_2 : \tau_{f_2} = I_2\alpha_2$$

$$\alpha_2 = \frac{2\mu_k M R g}{3I_2}$$

$$\omega_2(t) = \alpha_2 t$$

$$\omega_2(T) = \omega = \frac{I_1}{I_1+I_2}\omega_0 = \frac{2\mu_k M R g}{3I_2}T$$

$$\boxed{T = \frac{3I_1 I_2\omega_0}{2\mu_k M R g(I_1+I_2)}}$$

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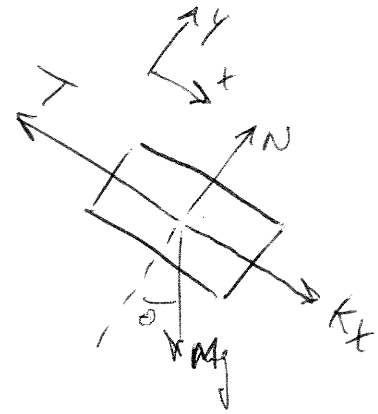
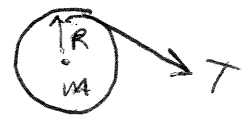




$\Sigma F_x: \; Kx + Mg\sin\theta - T = Ma \qquad \Sigma \tau_{cm} = TR = \left(\frac{1}{2}mR^2\right)\alpha$

Linear acceleration of block is the same as the tangential acceleration of the edge of the pulley, assuming a no-slip condition.

$$a = \alpha R \implies \alpha = \frac{a}{R}$$

$$TR = \frac{1}{2}mR^2\frac{a}{\cancel{R}}$$

$$T = \frac{1}{2}ma$$

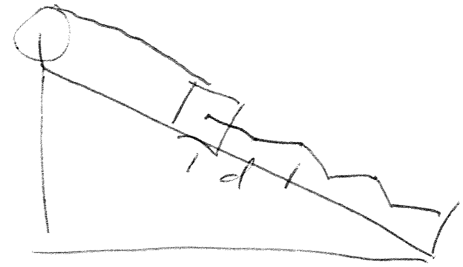
$$Kx + Mg\sin\theta - \frac{1}{2}ma = Ma$$

$$Kx + Mg\sin\theta = a\left(M + \frac{m}{2}\right)$$

$$a = \frac{Kx + Mg\sin\theta}{\left(M + \frac{m}{2}\right)}$$

$$\boxed{\alpha = \frac{Kx + Mg\sin\theta}{R\left(M + \frac{m}{2}\right)}}$$

$$E_i = Mgd\sin\theta + \tfrac{1}{2}Kd^2$$



$$E_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

Velocity of the block equals to tangential velocity of the edge of the pulley.

$$v = \omega R$$

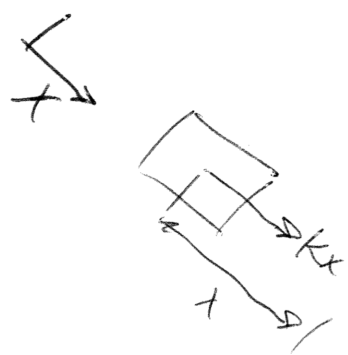
$$E_f = \tfrac{1}{2}M(\omega R)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\omega^2$$

$$= \tfrac{1}{2}MR^2\omega^2 + \tfrac{1}{4}mR^2\omega^2 = \tfrac{1}{2}R^2\omega^2\left(M + \tfrac{1}{2}m\right)$$

$$E_f = E_i$$

$$\tfrac{1}{2}R^2\omega^2\left(M + \tfrac{1}{2}m\right) = Mgd\sin\theta + \tfrac{1}{2}Kd^2$$

$$\boxed{\omega^2 = \frac{2Mgd\sin\theta + Kd^2}{R^2\left(M + \tfrac{1}{2}m\right)}}$$



Weight of the mass compressed the spring from its natural equil. length to a new equil. length. These forces cancel out.

$$\Sigma F_{ref}: \; Kx = K_{eff}\,x$$

$$K_{eff} = K$$

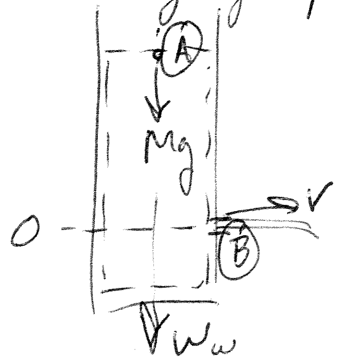
$$\omega^2 = \frac{K}{m} = \frac{K}{mg\cos\theta}$$

$$x(t) = d\cos\left(\frac{K}{mg\cos\theta}t\right)$$

(6)

If you assume pressure is due to the weight of the mass M only

In guage pressure



$$P_A = \frac{Mg}{A}$$

$$\frac{Mg}{A} + \rho g h + \frac{1}{2}\rho V_A^2 = \frac{1}{2}\rho V_B^2$$

from continuity $\quad A_A V_A = A_B V_B$

$$V_A = \frac{A_B}{A_A} V_B$$

$$V_A = \frac{\frac{1}{10}\cancel{A_A}}{\cancel{A_A}} V_B = \frac{V_B}{10}$$

$$\frac{Mg}{A} + \rho g h = \frac{1}{2}\rho\left(V_B^2 - \frac{V_B^2}{100}\right)$$

$$\frac{Mg}{A} + \rho g h = \frac{99}{200}\rho V_B^2$$

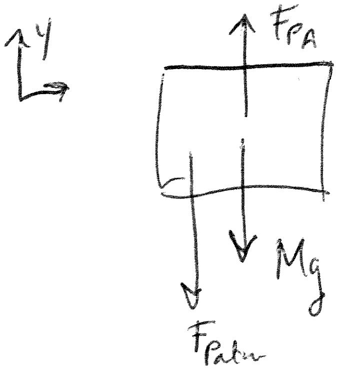
$$\boxed{V_B^2 = \frac{200}{99}\left[\frac{Mg}{\rho A} + g h\right]}$$

This would ~~be~~ be the velocity at the instant shown.

But there are problems with this solution.

If the mass falls at the same velocity as at A, it can be seen from the solution to V_B that velocity depends on how high the mass is from the hole. V

Force balance on mass M



$$\Sigma F_y: \; P_A A - P_{atm} A - Mg = Ma$$

$$P_A A - P_{atm} A - Mg = M\frac{dv_A}{dt} = \frac{M}{10}\frac{dv_B}{dt}$$

from Bernoulli's Equ. $P_A = P_{atm} - \rho g y + \frac{99}{200}\rho V_B^2$

where y is the distance from the hole to the bottom of mass M.

$$P_A = P_{atm} - \rho g y + \frac{99}{2}\rho v_A^2$$

substituting for P_A in the force balance

$$A\left[\cancel{P_{atm}} - \rho g y + \frac{99}{2}\rho\left(\frac{dy}{dt}\right)^2\right] - \cancel{P_{atm} A} - Mg = M\frac{dv_A}{dt} = M\frac{d^2y}{dt^2}$$

$$M\frac{d^2y}{dt^2} - \frac{99}{2}\rho A\left(\frac{dy}{dt}\right)^2 + \rho g A y - Mg = 0$$

This is a diff. eq. I don't think 7a students would be expected to solve.

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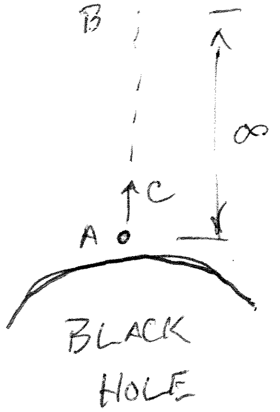


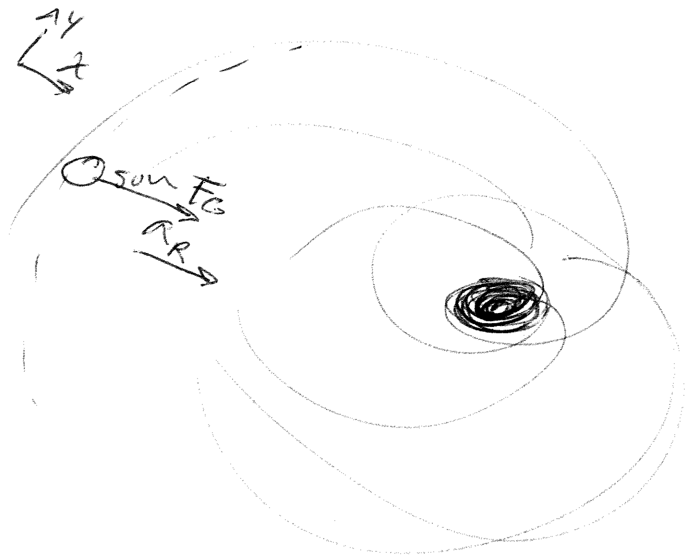
$$E_A = \frac{1}{2}mc^2 - \frac{GM_{BH}m}{R_s}$$

$$E_B = 0$$

$$\frac{1}{2}mc^2 - \frac{GM_{BH}m}{R_s} = 0$$

$$\boxed{R_s = \frac{2GM_{BH}}{c^2}}$$

$M_{BH} = 2.6 \times 10^6 M_S$



$$\Sigma F_x : \frac{GM_S M_G}{D^2} = M_S \frac{v^2}{D}$$

$$v = \frac{2\pi D}{T}$$

$$\boxed{T = \frac{2\pi D}{v}}$$

$$\boxed{M_G = \frac{v^2 D}{G} = \frac{4\pi^2}{GT^2} D^3}$$

$$M_{stars} = M_G - M_{BH}$$

$$\boxed{N_{stars} = \frac{M_{stars}}{M_{sun}} = \frac{M_G - M_{BH}}{M_{sol}}}$$